Graph theory – Problem set 1

1. Draw all graphs of order ≤ 4 and size 3, up to isomorphism.
2. $e(G) \leq C_2^n$ where $n = \text{ord}(G)$.
3. Draw $K_4, \overline{K_4}, P_4, C_4, C_5$.
4. In a bipartite graph, $e(G) \leq \lfloor \frac{n^2}{4} \rfloor$. The max is attained. For what graph ?
5. Devise an algorithm for building a spanning tree.
6. A tree of order n has size $n-1$.
   A forest of order n and k components has size $n-k$.
7. Let G be connected and $f : E(G) \to \mathbb{R}^+$ be a cost function. Find the most economical spanning sub-graph T of G.
8. Draw a Hamilton cycle in the dodecahedron.
9. Find a Hamilton cycle in the chess board with $E =$ knight moves.
10. Draw the graph representing Kônigsberg. Find a Euler circuit or trail.
11. Prove that K_5 and $K_{3,3}$ are non-planar.
12. (Euler formula) If a connected map has n vertices, m edges and f faces, then

$$\chi = n - m + f = 2.$$