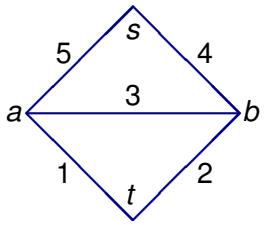
UNIVERSITÉ
de Cergy-Pontoise

Master M2 TPA
Graph Theory

Graph theory – Problem set 2



1. Find the current across the circuit (resistances in kΩ).
2. Find the total resistance across an edge of a cube with unit resistance along each edge.
3. Prove the star – triangle equivalence.
4. Let A be the adjacency, B the incidence, D the degree matrices of a graph. Check the relation (theorem 10)
$$B\,{}^tB = D - A$$
5. Show that the cycle space $Z(G) = \ker B$, where B is the incidence matrix.
6. Show that ${}^tB$ defines a linear mapping $C_0(G) \to C_1(G)$ and that $\operatorname{im} {}^tB = U(G)$, the cut space of G.
7. Given a set $F \subset E(G)$ of $n-1$ edges, let $\widetilde{B_F}$ be the $(n-1)\times(n-1)$ sub-matrix of B over F and $v_1, \ldots, v_{n-1}$. Show that $\widetilde{B_F}$ is invertible iff $F = E(T)$ for some tree T.
8. Finish the proof of theorem 9 for graphs with k components.