Quantum Mechanics 1 — PS 1

1 Quantum or not?

For each type of particle, e, n, p, compute the De Broglie wave length at temperature 300 K.
For each of these particles, what is the velocity corresponding to a wave length of 1 m?

2 Bohr quantization

From Bohr's quantization law,

$$\oint p\,dq = n\,h, \quad n = 0, 1, 2, \ldots,$$

get the energy levels E_n of

a) a harmonic oscillator of pulsation ω;

b) a particle in an infinitely deep potential well.

3 Gaussian wave function

Let φ be the wave function $\varphi(x) = C\exp\left(-\left[\frac{x-b}{2a}\right]^2\right)$.

a) Calculate C, $\langle x^n\rangle$ for $n = 1, 2, 3$ and $\Delta x = \langle (x - \langle x\rangle)^2\rangle^{1/2}$.

b) Calculate the Fourier transform $g(k)$ of φ.

c) Calculate $\langle k^n\rangle$ for $n = 1, 2, 3$ and Δk.

d) Calculate $\mathbb{P}(x > 0)$ and plot it as function of b.

4 Gaussian wave packet

Let $\psi(x, 0) = C\exp\left(-\left[\frac{x}{2a}\right]^2 + ik_0x\right)$ be a Gaussian quantum wave packet.

a) Calculate the amplitude of the k distribution, k being the wave vector.

b) Calculate $\psi(x, t)$ and the probability density $|\psi(x, t)|^2$, knowing the particle is free.

c) Calculate $\langle x\rangle(t)$ and show that the wave packet moves, on average, according the laws of classical mechanics.

d) Calculate the relative spread in the course of time. Comment.

5 Uncertainty

Show that the uncertainty Δx of a wave packet and that of its Fourier distribution Δk satisfy

$$\Delta x\,\Delta k \geq 1/2.$$